Lecture 7

Rules of Inferences (contd.), Introduction to Proofs

More Examples: Proving Validity of Arguments

Example: Prove that the following argument is valid.

Premises: $\begin{cases} (p \wedge q) \vee r \\ r \to s \end{cases}$ **Conclusion:** $p \vee s$

Solution:

Step	Reason
1. $(p \wedge q) \vee r$	Premise
2. $r \vee (p \wedge q)$	Commutative law on (1)
3. $(r \vee p) \wedge (r \vee q)$	Distributive law on (2)
4. $r \vee p$	Simplification of (3)
5. $r \to s$	Premise
6. $\neg r \vee s$	$\because p \to q \equiv \neg p \vee q$
7. $p \vee s$	Resolution on (4) and (6)

Allowed to use known laws.

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More Examples: Proving Validity of Arguments

Example: Prove that the following argument is valid.

Premises:
- Students who passed the course either scored well or had good attendance.
- Darsh did not have good attendance in the course.
- Darsh passed the course.

Conclusion: Darsh scored well in the course.

Solution: Set the domain as "students in the course".

$P(x) = x$ passed the course.

$S(x) = x$ scored well in the course.

$A(x) = x$ had good attendance in the course.

continue…

More Examples: Proving Validity of Arguments

Argument:

Premises:
$$\begin{cases} \forall x(P(x) \rightarrow S(x) \vee A(x)) \\ \neg A(Darsh) \\ P(Darsh) \end{cases}$$

Conclusion: $S(Darsh)$

Proving Validity:

Steps	Reason
1. $\forall x(P(x) \rightarrow S(x) \vee A(x))$	Premise
2. $P(Darsh) \rightarrow S(Darsh) \vee A(Darsh)$	Universal Instantiation using (1)
3. $P(Darsh)$	Premise
4. $S(Darsh) \vee A(Darsh)$	Modus Ponens using (2) and (3)
5. $\neg A(Darsh)$	Premise
6. $S(Darsh)$	Disjunctive Syllogism of (4) and (5)

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Introduction to Proofs

What's a proof?

In the broadest sense, **proof** is a method of establishing **truth**.

Truth can be established in more than one ways.

- In **experimental sciences** the truth is guessed and the hypothesis is confirmed or refuted by experiments.
- In **history** truth is obtained through archaeology, multiple historical texts, etc.
- In **day-to-day life** the truth is established by quoting scriptures, books, judiciary orders, etc.

What's a mathematical proof?

A mathematical proof is a sequence of statements, where each statement is either "logically supported" by the previous ones or is an already established truth or an axiom, that demonstrates the truth of a mathematical statement.

Introduction to Proofs

Mathematical proofs are of two types:

Formal Proofs

1. Axioms are properly stated.
2. Rules of inferences are explicitly stated.
3. Arranged as a sequence of statements.
4. One rule of inference is applied at every step.
5. Existing true statements/facts are not usually used.
6. Good for machines.

Informal Proofs

1. Axioms are sometimes skipped.
2. Rules of inferences are not explicitly stated.
3. Written like a mathematical essay.
4. Multiple rules of inferences can be applied in one step.
5. Existing true statements/facts are used.
6. Good for humans.

We will focus on **informal proofs** rather than **formal proofs**.